(3) Debye Theory (Quantum Theory) :-

Enstein theory assumed that N oscillators, oscillates with same natural freq. ω independently but as they are coupl there freq of vibration can not be same & also a solid can propogate a large no. of frequencies from low freq. to (sound waves) to high freq. (visible radiation). This means a crystal can have a no. of modes. And density of modes

$$\boxed{D(\nu) \propto \nu^2}$$

It is called Density of states.

Density of states of EM radiation $\Rightarrow$ $\boxed{D(E) \propto E^2}$

Density of state → No. of energy states available per unit energy interval. As $\nu \uparrow$, $D(\nu) \uparrow$

A crystal can propogate two types of wave :-

(i) Longitudnal Waves (Sound Waves) :- A longitudnal wave can have only 1 mode of vibration per harmonic oscillator. (propogation of wave along the dir^n of wave)

(ii) Transverse Wave :- A transverse wave have 2 mode of vibration $\perp^r$ to the dir^n of propogation.

specific heat from this theory comes out to be

$$\boxed{C_V = 3R\left(\frac{\theta_D}{T}\right)F_D}$$

$F_D \rightarrow$ Debye fun^n

$$C_V = 3R\left(\frac{\Theta_D}{T}\right)^3 3\left(\frac{T}{\Theta_D}\right)^4 \int_0^{\frac{\Theta_D}{T}} \frac{e^x x^4}{(e^x-1)^2}\, dx$$

where $\Theta_D \rightarrow$ Debye temp.

$\omega_D \rightarrow$ Debye ang. freq.

$$K_B \Theta_D = \hbar\omega_D = h\nu_D$$

$$\boxed{\Theta_D = \left(\frac{h\nu_D}{K_B}\right)}$$

It behaves as reference temp inside the crystal (solid)

Case:1 :- At High temp.

$$\hbar\omega_D = h\nu_D << K_B T$$

$$\boxed{\Theta_D << T}$$

for this region, $\int_0^{\Theta_E/T} \frac{e^x x^4}{(e^x-1)^2}\, dx = \frac{1}{3}\left(\frac{\Theta_D}{T}\right)^3$

$$\therefore \boxed{C_V = 3R}$$

At high temp., Debye theory matches with classical theory.

Case:2 :- At low temp. $\hbar\omega_D = h\nu_D >> K_B T$

$$\boxed{\Theta_D >> T}$$

$$\int_0^{\Theta_E/T} \frac{e^x x^4\, dx}{(e^x-1)^2} = \frac{\pi^4}{15}$$

$$\therefore \boxed{C_V = 12R\left(\frac{T}{\Theta_D}\right)^3 \frac{\pi^4}{15}}$$ Imp

At low temp. $C_V \propto \left(\frac{T}{\Theta_D}\right)^3 \Rightarrow \boxed{C_V \propto T^3}$

& this result matches with the experimental result.